Do not open this booklet until you are directed to do so.

1. Fill out completely the following information about yourself.

PRINT

<table>
<thead>
<tr>
<th>Last name</th>
<th>First name</th>
<th>Initial</th>
<th>Phone No.</th>
</tr>
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ADDRESS

<table>
<thead>
<tr>
<th>Street address</th>
<th>City</th>
<th>State</th>
<th>Zip</th>
</tr>
</thead>
</table>

Your high school: Name ___________________________ City ___________________________

High School Counselor or Advisor: ___________________________

2. This examination consists of two parts. The time allowed for each will be approximately 60 minutes. Should you finish Part I early, you may proceed to Part II.

3. Part I consists of 20 objective-type questions. Each question has five possible answers marked: A, B, C, D, E. Only one answer is correct. You are to circle the letter corresponding to the correct response for as many problems as you can.

Example: If $x = 5$ and $y = -2$, then $x + 4y$ is

A. $-3$  
B. $-2$  
C. $-1$  
D. $0$  
E. $+1$.

4. Part II consists of 3 subjective-type questions. Show a summary of your work in this booklet for each question you attempt, whether or not you obtain a complete solution. Scratch paper is provided but be sure to show the essential steps of your work concisely in the space provided for each question. Only the work appearing in this booklet will be scored. You will be scored on your method of attack, ingenuity, insight, inventiveness, and logical developments as well as your solutions.

5. Pencils and scratch paper will be provided. No tables, rulers, compasses, protractors, slide rules, calculators, or other aids are permitted.

6. a. The scoring of questions in Part I has been devised to discourage random guessing and will be computed as follows:

   
   (three times number correct) - (number wrong).

b. The scoring for the three questions in Part II will be 12, 12, and 16 for a total of 40 points. Partial credit will be given so it will be to your advantage to do as much as you are able to do on each question.

7. For the scoring committee. Do not write in the box below.

<table>
<thead>
<tr>
<th>Part I:</th>
<th>Part II:</th>
<th>Score on Part I:</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. Correct:</td>
<td>Score on 1:</td>
<td>Score on Part II:</td>
</tr>
<tr>
<td>No. Wrong:</td>
<td>Score on 2:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Score on 3:</td>
<td>TOTAL:</td>
</tr>
</tbody>
</table>
PART I

1. Last year a bicycle cost $160 and a cycling helmet cost $40. This year the cost of the bicycle increased by 5%, and the cost of the helmet increased by 10%. The percent increase in the combined cost of the bicycle and the helmet is

(A) 6%
(B) 7%
(C) 7.5%
(D) 8%
(E) 15%

2. Al and Barb start their jobs on the same day. Al's schedule is 3 work-days followed by 1 rest-day. Barb's schedule is 7 work-days followed by 3 rest-days. On how many of their first 1000 days do both have rest-days on the same day?

(A) 48
(B) 50
(C) 72
(D) 75
(E) 100

3. In the $xy$-plane, the segment with endpoints $(-5,0)$ and $(25,0)$ is the diameter of a circle. If the point $(x,15)$ is on the circle, then $x =$

(A) 10
(B) 12.5
(C) 15
(D) 17.5
(E) 20

4. Points $A, B,$ and $C$ are on a circle of radius $r$ so that $AB = AC$, $AB > r$, and the length of minor arc $BC$ is $r$. If angles are measured in radians, then $AB/BC =$

(A) $\frac{1}{2} \csc \frac{1}{2}$
(B) $2 \cos \frac{1}{2}$
(C) $4 \sin \frac{1}{2}$
(D) $\csc \frac{1}{2}$
(E) $2 \sec \frac{1}{2}$
5. The sum of the series

\[ 20 + 20 \frac{1}{5} + 20 \frac{2}{5} + \ldots + 40 \]

is

(A) 3000
(B) 3030
(C) 3150
(D) 4100
(E) 6000

6. If \( a \) and \( b \) are positive numbers such that \( a^b = b^a \) and \( b = 9a \), then the value of \( a \) is

(A) 9
(B) \( \frac{1}{9} \)
(C) \( \sqrt[3]{3} \)
(D) \( \sqrt[9]{9} \)
(E) \( \sqrt[3]{3} \)

7. If \( \log_7(\log_3(\log_2 x)) = 0 \), then \( x^{-1/2} = \)

(A) \( \frac{1}{3} \)
(B) \( \frac{1}{2\sqrt{3}} \)
(C) \( \frac{1}{3\sqrt{3}} \)
(D) \( \frac{1}{\sqrt{42}} \)
(E) None of these.

8. The increasing sequence of positive integers \( a_1, a_2, \ldots \) has the property that

\[ a_{n+2} = a_n + a_{n+1} \text{ for all } n \geq 1. \]

If \( a_7 = 120 \), then \( a_8 \) is

(A) 128
(B) 168
(C) 193
(D) 194
(E) 210
9. If \[ f \left( \frac{x}{x - 1} \right) = \frac{1}{x} \text{ for all } x \neq 0, 1 \]
and \( 0 < \theta < \frac{\pi}{2} \), then \( f(\sec^2 \theta) = \)

(A) \( \sin^2 \theta \)
(B) \( \cos^2 \theta \)
(C) \( \tan^2 \theta \)
(D) \( \cot^2 \theta \)
(E) \( \csc^2 \theta \)

10. The sides of \( \triangle ABC \) have lengths 6, 8, and 10 with \( \angle C = 90^\circ \). A circle with center \( P \) and radius 1 rolls around the inside of \( \triangle ABC \), always remaining tangent to at least one side of the triangle. Through what distance has point \( P \) traveled?

(A) 10
(B) 12
(C) 14
(D) 15
(E) 17

11. If \( x \geq 0 \), then \( \sqrt[3]{x} \sqrt{x} \sqrt[3]{x} = \)

(A) \( x \sqrt[3]{x} \)
(B) \( x \sqrt{x} \)
(C) \( \sqrt[3]{x} \)
(D) \( \frac{\sqrt[3]{x^3}}{x} \)
(E) \( \frac{\sqrt{x^7}}{\sqrt{x}} \)

12. Country \( A \) has \( c\% \) of the world’s population and owns \( d\% \) of the world’s wealth. Country \( B \) has \( e\% \) of the world’s population and \( f\% \) of its wealth. Assume that the citizens of \( A \) share the wealth of \( A \) equally, and assume that those of \( B \) share the wealth of \( B \) equally. Find the ratio of the wealth of a citizen of \( A \) to the wealth of a citizen of \( B \).

(A) \( \frac{cd}{ef} \)
(B) \( \frac{ce}{df} \)
(C) \( \frac{cf}{de} \)
(D) \( \frac{de}{cf} \)
(E) \( \frac{df}{ce} \)
13. Let \( a, b, c, \) and \( d \) be integers with \( a < 2b, b < 3c, \) and \( c < 4d. \) If \( d < 100, \) the largest possible value for \( a \) is

\[ \begin{align*}
(A) & \; 2367 \\
(B) & \; 2375 \\
(C) & \; 2391 \\
(D) & \; 2399 \\
(E) & \; 2400
\end{align*} \]

14. A lattice point is a point in the plane with integer coordinates. How many lattice points are on the line segment whose endpoints are \((3,17)\) and \((48,281)\)? (Include both endpoints of the segment in your count.)

\[ \begin{align*}
(A) & \; 2 \\
(B) & \; 4 \\
(C) & \; 16 \\
(D) & \; 26 \\
(E) & \; 46
\end{align*} \]

15. Let \( x \) be a real number selected uniformly at random between 100 and 200. If \( \sqrt[3]{x} = 12 \) find the probability that \( \lfloor \sqrt{100x} \rfloor = 120. \) (\( \lfloor v \rfloor \) means the greatest integer less than or equal to \( v \).)

\[ \begin{align*}
(A) & \; 0 \\
(B) & \; \frac{241}{2500} \\
(C) & \; \frac{1}{10} \\
(D) & \; \frac{96}{625} \\
(E) & \; 1
\end{align*} \]

16. Nine congruent spheres are packed inside a unit cube in such a way that one of them has its center at the center of the cube and each of the others is tangent to the center sphere and to three faces of the cube. What is the radius of each sphere?

\[ \begin{align*}
(A) & \; 1 - \frac{\sqrt{3}}{2} \\
(B) & \; \frac{2\sqrt{3} - 3}{2} \\
(C) & \; \frac{\sqrt{2}}{6} \\
(D) & \; \frac{1}{4} \\
(E) & \; \frac{\sqrt{3}(2 - \sqrt{2})}{4}
\end{align*} \]
17. Ten people form a circle. Each picks a number and tells it to the other two neighbors adjacent to him in the circle. Then each person computes and announces the average of the numbers of his two neighbors. The figure on the right shows the average announced by each person (not the original number that the person picked). The average of the numbers drawn by all ten people is

(A) 5
(B) 1
(C) 6
(D) 5.5
(E) Not uniquely determined from the given information.

18. Let $M$ be the midpoint of $AB$. Segments $MP$ and $MQ$ are parallel to $BC$ and $AC$ respectively. If $AB = 3$, $AC = 4$, $BC = 5$, then the ratio $\frac{MP}{MQ} =$

(A) 1
(B) 1.25
(C) 1.5
(D) 1.75
(E) 2

19. Convex pentagon $ABCDE$ has $\angle A = \angle B = 120^\circ$, $EA = AB = BC = 2$ and $CD = DE = 4$. What is the area of $ABCDE$?

(A) 10
(B) $7\sqrt{3}$
(C) 15
(D) $9\sqrt{3}$
(E) $12\sqrt{5}$

20. An 8 by $2\sqrt{2}$ rectangle has the same center as a circle of radius 2. The area of the region common to both the rectangle and the circle is

(A) $2\pi$
(B) $2\pi + 2$
(C) $4\pi - 4$
(D) $2\pi + 4$
(E) $4\pi - 2$
PART II

1. Let \( p(x) = x^2 + bx + c \), where \( b \) and \( c \) are integers. If \( p(x) \) is a factor of both

\[
x^4 + 6x^2 + 25 \quad \text{and} \quad 3x^4 + 4x^2 + 28x + 5,
\]

what is \( p(1) \)?
2. Initially an urn contains 100 black marbles and 100 white marbles. Repeatedly, three marbles drawn at random are removed from the urn and replaced from a pile outside the urn as follows:

<table>
<thead>
<tr>
<th>Marbles Removed</th>
<th>Replaced with</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 black</td>
<td>1 black</td>
</tr>
<tr>
<td>2 black, 1 white</td>
<td>1 black, 1 white</td>
</tr>
<tr>
<td>1 black, 2 white</td>
<td>2 white</td>
</tr>
<tr>
<td>3 white</td>
<td>1 black, 1 white</td>
</tr>
</tbody>
</table>

The drawing process stops if less than three marbles are left in the urn. Explain what marbles will be left at the end. You must consider all possibilities.
3. Find the largest value of $\frac{y}{x}$ for pairs of real numbers $(x, y)$ which satisfy 

$$(x - 3)^2 + (y - 3)^2 = 6.$$  

16 POINTS