Do not open this booklet until you are directed to do so.

1. Fill out completely the following information about yourself.

   PRINT
   Last name  First name  Initial  Phone No.

   ADDRESS
   Street address  City  State  Zip

   Your high school: Name ___________________________ City ___________________________
   High School Counselor or Advisor: ________________________________________________

2. This examination consists of two parts. The time allowed for each will be approximately 60 minutes. Should you finish Part I early, you may proceed to Part II.

3. Part I consists of 15 objective-type questions. Each question has five possible answers marked: A., B., C., D., E. Only one answer is correct. You are to circle the letter corresponding to the correct response for as many problems as you can.

   Example: If \( x = 5 \) and \( y = -2 \), then \( x + 4y \) is
   \[ \begin{array}{c}
   \text{A.} -3 \\
   \text{B.} -2 \\
   \text{C.} -1 \\
   \text{D.} 0 \\
   \text{E.} +1.
   \end{array} \]

4. Part II consists of 3 subjective-type questions. Show a summary of your work in this booklet for each question you attempt, whether or not you obtain a complete solution. Scratch paper is provided but be sure to show the essential steps of your work concisely in the space provided for each question. Only the work appearing in this booklet will be scored. You will be scored on your method of attack, ingenuity, insight, inventiveness, and logical developments as well as your solutions.

5. Pencils and scratch paper will be provided. No tables, rulers, compasses, protractors, slide rules, calculators, or other aids are permitted.

6. a. The scoring of questions in Part I has been devised to discourage random guessing and will be computed as follows:

   \[ \text{Score on Part I: } (\text{four times number correct}) - \text{(number wrong)}. \]

   b. The scoring for the three questions in Part II will be 13, 13, and 14 for a total of 40 points. Partial credit will be given so it will be to your advantage to do as much as you are able to do on each question.

7. For the scoring committee. Do not write in the box below.

<table>
<thead>
<tr>
<th>Part I:</th>
<th>Part II:</th>
<th>Score on Part I:</th>
</tr>
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<tbody>
<tr>
<td>No. Correct:</td>
<td>Score on 1:</td>
<td>Score on Part I:</td>
</tr>
<tr>
<td>No. Wrong:</td>
<td>Score on 2:</td>
<td>Score on Part II:</td>
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<tr>
<td>Score on 3:</td>
<td>TOTAL:</td>
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</table>
1. \( P \) is a point inside the square \( ABCD \). The distances from \( P \) to the sides \( AB, BC, CD \) and \( DA \) are 1, 3, 7 and \( d \), respectively. Then \( d \) equals

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

2. The side \( BC \) of the rectangle \( ABCD \) has length 6. The circle which passes through the points \( A, B \) and \( C \) has a radius of length 5. Then \( CD \) has length

(A) 7

(B) 8

(C) 6

(D) 5

(E) 9

3. If \( x - y = 5 \) and \( xy^2 + 6xy - x^2y = 1 \), then \( xy \) equals

(A) \(-1\)

(B) 0

(C) 1

(D) 2

(E) 3

4. If \( a, b, c \) are integers such that \( \frac{a + c}{b} = \frac{2}{3} \) and \( \frac{b}{a - c} = \frac{3}{5} \), then \( \frac{a}{c} \) equals

(A) \( \frac{2}{5} \)

(B) \( -\frac{5}{2} \)

(C) \( \frac{7}{6} \)

(D) \( -\frac{7}{3} \)

(E) \( \frac{3}{7} \)
5. If the measures of the interior angles of a triangle are $a, b$ and $c$, where $a < b < c$, then

(A) $b + c > \frac{2\pi}{3}$

(B) $c \geq \frac{\pi}{2}$

(C) $c - b \geq \frac{\pi}{12}$

(D) $b = \frac{\pi}{3}$

(E) $a + b < c$

6. The edges of cube $C_1$ have length $r_1$, the edges of cubes $C_2$ have length $r_2$. If

\[
\frac{\text{volume } C_1}{\text{volume } C_2} \text{ equals twice } \frac{\text{surface area } C_1}{\text{surface area } C_2}
\]

then

(A) $r_1 = \sqrt{2}r_2$

(B) $r_2 = \sqrt{2}r_1$

(C) $r_2 = \frac{1}{2}r_1$

(D) $r_1 = \sqrt{2}r_2$

(E) $r_1 = r_2$

7. If $\tan x = 4$ then $\sin x \cos x$ equals

(A) $\frac{2}{17}$

(B) $\frac{3}{17}$

(C) $\frac{4}{17}$

(D) $\frac{5}{17}$

(E) $\frac{6}{17}$

8. $\frac{1}{\sqrt{10} - \sqrt{14}}$ equals

(A) $\frac{1}{2}$

(B) $-\frac{1}{2}$

(C) $\frac{\sqrt{10} - \sqrt{14}}{2}$

(D) $-\frac{\sqrt{14} + \sqrt{10}}{2}$

(E) $-\frac{\sqrt{10} + \sqrt{14}}{4}$
9. If $2a + 3b + c = 15$ and $3a + 2b - c = 25$ then
   
   (A) $a + b = 1$
   (B) $b + c = -1$
   (C) $a > 0$
   (D) $a = b$
   (E) $a = 4, b = 4, c = -5$

10. Let $a > 0, b > 0, a \neq 1, b \neq 1$, be such that

   \[
   \left( \log_a b \right) \left( \log_{\frac{1}{b}} \frac{1}{a} \right) = 1.
   \]

   Then
   
   (A) $a = b$
   (B) $a = \frac{1}{b}$
   (C) $a > 1$
   (D) $ab = 1$ or $a = b$
   (E) $\frac{1}{a} + \frac{1}{b} = 1$

11. If $125^x = 0.0016$, then $x$ equals

   (A) $\frac{1}{4}$
   (B) $\frac{3}{4}$
   (C) $\frac{4}{3}$
   (D) $-\frac{4}{3}$
   (E) $\frac{1}{24}$

12. Let the point $(x, y)$ be on the graph of $y = \sqrt{x}$. Then

   (A) $x > 0$
   (B) $(x, y)$ cannot be on the graph of $x = 1$
   (C) $(x, y)$ cannot be on the graph of $y = 2$
   (D) $x$ and $y$ can both be fractions
   (E) $x - y > 0$
13. If the point \( \left( \frac{2\sqrt{2}}{3}, \frac{2\sqrt{2}}{3} \right) \) belongs to the ellipse \( \frac{x^2}{a^2} + y^2 = 1, \ a > 0, \) then \( a \) equals

(A) \( \frac{1}{\sqrt{8}} \)

(B) \( \sqrt{2} \)

(C) \( \frac{1}{\sqrt{2}} \)

(D) \( \frac{2}{\sqrt{8}} \)

(E) \( 2\sqrt{2} \)

14. If

\[
\left( 1 + \frac{1}{2} x + 2x^3 \right)^3 = 1 + a_1 x + a_2 x^2 + \ldots + a_8 x^8 + a_9 x^9
\]

then \( a_4 \) equals

(A) 1

(B) 8

(C) 4

(D) \( \frac{1}{8} \)

(E) 6

15. The sequence \( a_1, a_2, a_3, \ldots \) is a geometric progression such that \( a_1 > 0, \ a_4 = -\frac{1}{4} \) and \( a_8 = -\frac{1}{64}. \) Then \( a_3 \) equals

(A) \( \frac{1}{8} \)

(B) \(-4\)

(C) 2

(D) \( \frac{1}{2} \)

(E) \( -\frac{1}{2} \)
PART II

1. Every cube has 6 faces and 8 vertices

(a) In how many ways can one select three vertices which are in the same face?

(b) In how many ways can one select three vertices which are not in the same face?
2. The sides of the square $ABCD$ have length 1. The points $E, F, G, H$ are inside this square and the triangles $ABE$, $BCF$, $CDG$ and $DAH$ are equilateral.  

(a) Give a sketch where all the mentioned points are appropriately labeled.

(b) Find the length $a + b\sqrt{3}$ of $EG$ ($a$ and $b$ integers).

(c) Find the area $c + d\sqrt{3}$ of the square $EFGH$ ($c$ and $d$ integers).
3. For every positive integer $n$, put

$$A_n = \frac{1}{10} + \frac{1}{10^2} + \ldots + \frac{1}{10^n}$$

and

$$\frac{1}{9} = A_n + \frac{1}{B_n}$$

(a) Write $A_5$ in decimal notation and write $A_5$ as a fraction.

(b) $B_n$ is an integer: find $B_5$.

(c) Find the smallest positive integer $n$ such that $\frac{1}{9} - A_n < 10^{-99}$. 