Marquette University Executive MBA Program
Mathematics Review Course
Workbook Summer 2024

Chapter Zero: Algebraic Concepts

Play Section 0.2 Discussion 1

Section 0.2: The Real Numbers

Natural Numbers { 1, 2, 3, ...}

Integers { ... -3, -2, -1, 0, 1, 2, 3, ...}

Rational Numbers $\frac{a}{b}$ where a and b are integers and $b \neq 0$

Irrational Numbers non-terminating, non-repeating decimals

Real Numbers rational and irrational numbers

Try the following:
2 p.14

Play Section 0.2 Exercises 1 and Section 0.2 Discussion 2
Not at the same time… that would be confusing

Properties of Real Numbers

Commutative property: Addition: $a+b=b+a$
Multiplication: $ab=ba$

Associative property: Addition: $(a+b)+c=a+(b+c)$
Multiplication: $(ab)c=a(bc)$

Identity: Additive identity is 0: $a+0=0+a=a$
Multiplicative identity is 1: $a1=1a=a$

Inverse:

Each element a has an additive inverse, denoted $-a$:

$$a+(-a)=-a+a=0$$

Each nonzero element a has a multiplicative inverse, denoted by a^{-1}

$$aa^{-1}=a^{-1}a=1$$

note that $a^{-1}=\frac{1}{a}$

Distributive property: $a(b+c)=ab+ac$

Interval Notation

Open interval: (a,b) means $a<x<b$ is the solution set

Closed interval: $[a,b]$ means $a\le x\le b$ is the solution set

Try the following:
28 p. 14

30 p. 14

32 p. 14

34 p. 14

36 p. 14

Play Section 0.2 Exercises 2 and Section 0.2 Discussion 3

The **Absolute Value** of a, written $|a|$, is $|a|$ = a if $a\ge 0$
= $-a$ if $a<0$

Order of Operations

1. Perform operations in parentheses.
2. Find indicated powers.
3. Perform multiplications and divisions left to right.
4. Perform additions and subtractions left to right.

Try the following:
12 p. 14

16 p. 14

Play Section 0.2 Exercises 3

Section 0.3: Integral Exponents

Play Section 0.3 Discussion 1

If n is a positive integer:

1. $a^n = aaa \cdots a$ (n times)
2. $a^{-n} = \dfrac{1}{a^n}$
3. $\dfrac{1}{a^{-n}} = a^n$
4. $a^0 = 1$ if $a \neq 0$. 0^0 is undefined.

Rules of Exponents:

1. $a^m a^n = a^{m+n}$
2. $\dfrac{a^m}{a^n} = a^{m-n} = \dfrac{1}{a^{n-m}}$
3. $(ab)^m = a^m b^m$ $\qquad \left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$
4. $(a^m)^n = a^{mn}$
5. $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$

Try the following:

2b p. 19 $\qquad$ 4a p. 19

4b p. 19 $\qquad$ 12 p. 19

14 p. 19　　　　16 p. 19

18 p. 19　　　　20 p. 19

22 p. 19　　　　24 p. 19

30 p. 19　　　　36 p. 19

40 p. 19

42 p. 19

Play Section 0.3 Exercises 1

Section 0.4: Radicals and Rational Exponents

Play Section 0.4 Discussion 1

If $b^n = a$, where n is a positive integer, b **is an** n^{th} **root of** a.

The **Principal** n^{th} **root of** a is that n^{th} root of a which is positive if a is positive and negative if a is negative.

So $\sqrt[n]{a}$ is positive if a is positive.
negative if a is negative and n is odd.

$\sqrt[n]{a}$ is called a **Radical.** $\sqrt{\ }$ is the **Radical Sign.**

For a positive integer n, $a^{1/n} = \sqrt[n]{a}$ if $\sqrt[n]{a}$ exists.

So, for positive integer n and integer m,

$a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$.　　(If n is even, a must be nonnegative)

Rules for Radicals

Given that $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real (if n is even, $a \geq 0$ and $b \geq 0$),

1. $\sqrt[n]{a^n} = (\sqrt[n]{a})^n = a$

2. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

3. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$ assuming $b \neq 0$

Try the following:
1ab p. 26

2 p. 26

8 p. 26

18 p. 26

20 p. 26

24 p. 26

32 p. 26

58 p. 26

72 p. 27

Play Section 0.4 Exercises 1

Section 0.5: Operations with Algebraic Expressions

Play Section 0.5 Discussion 1

Algebraic Expression An expression obtained by performing the operations of addition, subtraction, multiplication, division, exponents or roots.

Polynomial in x An algebraic expression of the form

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$$

where n is a nonnegative integer and the as are constants with $a_n \neq 0$.

n is called the **Degree** of the polynomial.

Special Products

A. $(x+a)(x+b) = x^2 + (a+b)x + ab$

B. $(ax+b)(cx+d) = acx^2 + (ad+bc)x + bd$

C. $(x+a)^2 = x^2 + 2ax + a^2$

D. $(x-a)^2 = x^2 - 2ax + a^2$

E. $(x+a)(x-a) = x^2 - a^2$

F. $(x+a)^3 = x^3 + 3ax^2 + 3a^2x + a^3$

G. $(x-a)^3 = x^3 - 3ax^2 + 3a^2x - a^3$

Try the following:

16 p. 32

24 p. 32

26 p. 32

30 p. 32

34 p. 32

Play Section 0.5 Exercises 1

Try the following:
70 p. 33

Play Section 0.5 Exercises 2

Section 0.6: Factoring
Play Section 0.6 Discussion 1
Use above special products backwards.

examples, see C - E above

$$x^2 + 2ax + a^2 = (x + a)^2$$

$$x^2 - 2ax + a^2 = (x - a)^2$$

$$x^2 - a^2 = (x + a)(x - a)$$

Try the following:
2b p. 37

6 p. 37

8 p. 37

16 p. 37

Play Section 0.6 Exercises 1

Section 0.7: Algebraic Fractions

Try the following:
2 p. 43

4 p. 43

8 p. 43

16 p. 43

22 p. 43

24 p. 43

Play Section 0.7 Exercises 1

Homework: Review Exercises (pages 46-49) do as many of 8-18, 19a,20-31,33-42,45-58,61-84,88,90,96,98 as needed to feel comfortable with the material

Chapter One: Linear Equations and Functions

Play Section 1.1 Discussion 1

Section 1.1: Solution of Linear Equations in One Variable

Equation a statement that two expressions are equal.

To solve an equation is to find all values of its variables for which the equation is true. The values are called **Solutions.**

In solving, we want any operation on the equation to result in another equation with exactly the same solutions as (to be equivalent to) the given equation.

Operations that Guarantee Equivalence

1. Replace either side of the equation by an equal expression.

2. Add/Subtract the same quantity to/from both sides of the equation.

3. Multiply/Divide both sides by the same nonzero constant.

A **Linear Equation in x** is an equation that can be written in the form $ax+b=0$ where a and b are constants and $a\neq 0$.

If an equation has no solution, we call the solution set the **Empty Set** or **Null Set**.

Try the following:

2 p. 60

14 p. 60

Play Section 1.1 Exercises 1

Play Section 1.1 Discussion 2

Linear Inequalities if $a<b$

1. $a+c<b+c$ and $a-c<b-c$

2. and $c>0$, $ac<bc$ and $\frac{a}{c}<\frac{b}{c}$

3. and $c<0$, $ac>bc$ and $\frac{a}{c}>\frac{b}{c}$

4. and $a=c$, then $c<b$

5. If $0<a<b$, $\frac{1}{a}>\frac{1}{b}$

Try the following:
34 p. 60

62 p. 61

Play Section 1.1 Exercises 2

Section 1.2: Functions

Play Section 1.2 Discussion 1

A **Rectangular (Cartesian) Coordinate System** allows one to specify and locate points in a plane.

A **Relation** is defined by a set of ordered pairs or by a rule that determines how the ordered pairs are found.

A **Function** is a rule that assigns to each input number exactly one output number. The set of all input numbers to which the rule applies is called the **Domain** of the function. The set of all output numbers is called the **Range.**

A variable representing the input numbers is called an **Independent Variable.** A variable representing the output numbers is called the **Dependent Variable.**

Usual function notation: $f(x)$ means the output number in the range of f that corresponds to the input number, x, in the domain.

The **Graph** of a relation is the picture that is drawn by plotting the points whose coordinates (x,y) satisfy the relation.

Vertical Line Test If you can draw a vertical line that passes through two or more points of a graph, the graph is <u>not</u> the graph of a function.

Try the following:

5b p. 71

6b p. 71

14 p. 71

22 p. 71

26 p. 72

28 p. 72

30 p. 72

32 p. 72

Play Section 1.2 Exercises 1

Section 1.3: Linear Functions

Play Section 1.3 Discussion 1

A **Linear Function** is a function of the form $y = f(x) = ax + b$, where a and b are constants.

The graph of a linear function is always a straight line, so we need only two points to graph it.

The **x-intercept** is where the graph intersects the x-axis.

The **y-intercept** is where the graph intersects the y-axis.

Try the following:
2 p. 82

Play Section 1.3 Exercises 1 and Section 1.3 Discussion 2

Let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ be two points on a line where $x_1 \neq x_2$. The **Slope** of the line is the number m given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

If slope = 0, horizontal line
> 0, line rises left to right
< 0, line falls left to right

If $x_1 = x_2$, slope is undefined, vertical line

Two distinct lines are **Parallel** if and only if their slopes are equal or they are vertical.

Point-Slope Form of a line: $y - y_1 = m(x - x_1)$
is the line with slope m passing through the point (x_1, y_1).

Slope-Intercept Form of a line: $y = mx + b$
is the line with slope m and y-intercept b.

Try the following:
6 p. 82

18 p. 83

22 p. 83

24 p. 83

26 p. 83

30 p. 83

Play Section 1.3 Exercises 2

Section 1.5: Solutions of Systems of Linear Equations

Play Section 1.5 Discussion 1

Solutions of Systems of Linear Equations find the point(s) where two lines intersect.

Try the following:
8 p. 101

10 p. 101

6 p. 101

38 p. 102

32 p. 101

Play Section 1.5 Exercises 1

Section 1.6: Applications of Functions in Business and Economics

Play Section 1.6 Discussion 1

Fixed Cost The sum of all costs that are independent of the level of production.

Variable Cost The sum of all costs dependent on the level of production.

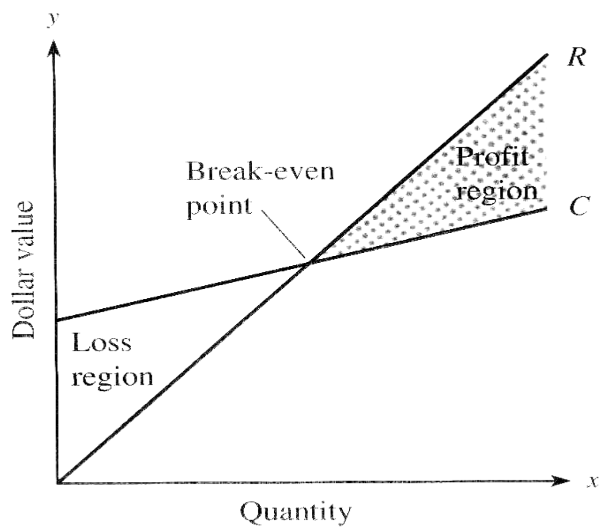
Total Cost = Fixed Cost + Variable Cost

Total Revenue = (price per unit)(number of units sold)

Profit = Total Revenue - Total Cost

Break-Even Point where $P(x)=R(x)-C(x)=0$ so $R(x)=C(x)$.

$R(x)=C(x)$ is break-even point
$R(x)>C(x)$ is profit region
$R(x)<C(x)$ is loss region



Try the following:
4 p. 109

Play Section 1.6 Exercises 1 and Section 1.6 Discussion 2

Marginal Cost the cost associated with producing the next unit.

the rate of change in total cost with respect to the number of units produced.

if the cost is linear, the slope of cost is the marginal cost.

Try the following:
6 p. 109

Play Section 1.6 Exercises 2 and Section 1.6 Discussion 3

Demand Equation relates p and q where:

p is the price (per unit) of a product.

q is the corresponding quantity of that product that consumers will demand at price p.

Law of Demand: as price increases, quantity demanded will fall.

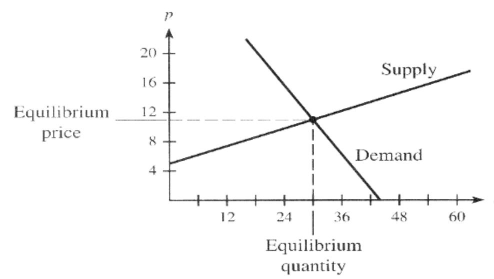
Supply Equation relates p and q where:

p is the price of a product.

q is the corresponding quantity of that product the producers are willing to supply at price p.

Law of Supply: as price rises, the corresponding quantity supplied will also rise.

When supply and demand curves are represented on the same coordinate system, the point where the curves intersect is called the **Point of Equilibrium.**



Try the following:
34 p. 112

46 p. 112

Play Section 1.6 Exercises 3

Homework: Review Exercises (pages 114-118) Try as many of 1-16, 18-22,24-35,45-51,53,54,61,62,66,68-71 as needed to feel comfortable with the material

Chapter Two: Quadratic Equations

Play Section 2.1 Discussion 1

Section 2.1: Quadratic Equations

A **Quadratic Equation** in x is a second-degree polynomial equation in x. It can be written in the form $ax^2+bx+c=0$, where a, b, and c are constants and $a \neq 0$.

The easiest quadratic equations to solve are those that factor nicely.

Set one side of the equation equal to zero then use the fact that $ab=0$ if and only if $a=0$ or $b=0$ or both.

Try the following:
8 p. 129

10 p. 129

28 p. 129

Play Section 2.1 Exercises 1 and Section 2.1 Discussion 2

If C is a nonnegative constant, the solution of $x^2=C$ is $x=\pm\sqrt{C}$.

Try the following:
8 p. 129

22 p. 129

Play Section 2.1 Exercises 2

Play Section 2.1 Discussion 3

If neither of the above works, try the **Quadratic Formula:**

If $ax^2 + bx + c = 0$ and $a \neq 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

If $b^2 - 4ac$ < 0, no real solutions
> 0, two distinct real solutions
= 0, one real solution

Try the following:

16 p. 129

Play Section 2.1 Exercises 3

Section 2.3: Business Applications of Quadratic Functions

Try the following:

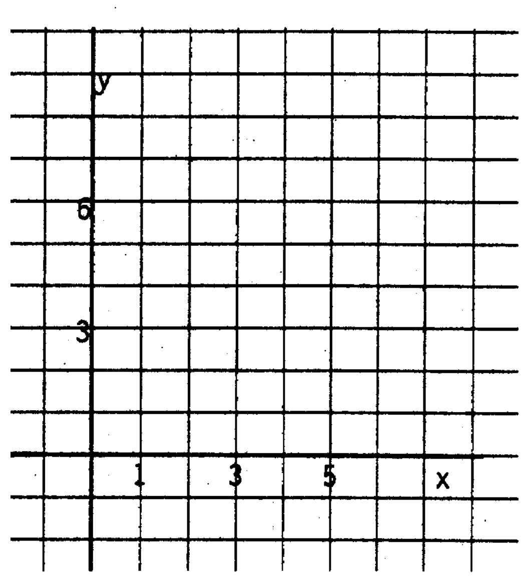
4 p. 146

Homework: Review Exercises (pages 171-173) Try as many of 1-10,49,55-58 as needed to feel comfortable with the material.

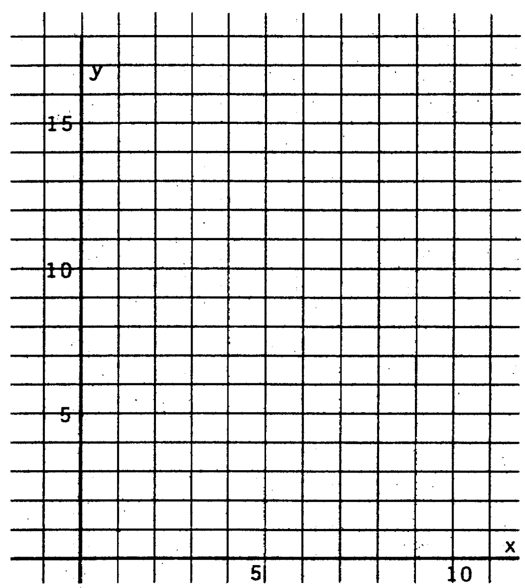
Chapter Four: Inequalities and Linear Programming

Section 4.1: Linear Inequalities in Two Variables

Try the following:
16 p. 258



18 p. 258 also find the corner points



Play Section 4.1 Exercises 1

Section 4.2: Linear Programming: Graphical Methods

Play Section 4.2 Discussion 1

Linear Programming: Graphical Methods

We will try to maximize (or minimize) a linear function in x and y ($f = ax + by$) subject to some constraints.

The function to be maximized or minimized is the **Objective Function.**

The possible solutions to the system of constraints are called **Feasible Solutions.**

The **Optimum Solution** is the feasible solution maximizing or minimizing the objective function.

See graph on previous page

Maximize $f = 2x + y$ subject to:

$$-x + y \leq 2$$
$$x + 2y \leq 10$$
$$3x + y \leq 15$$
$$x \geq 0, y \geq 0$$

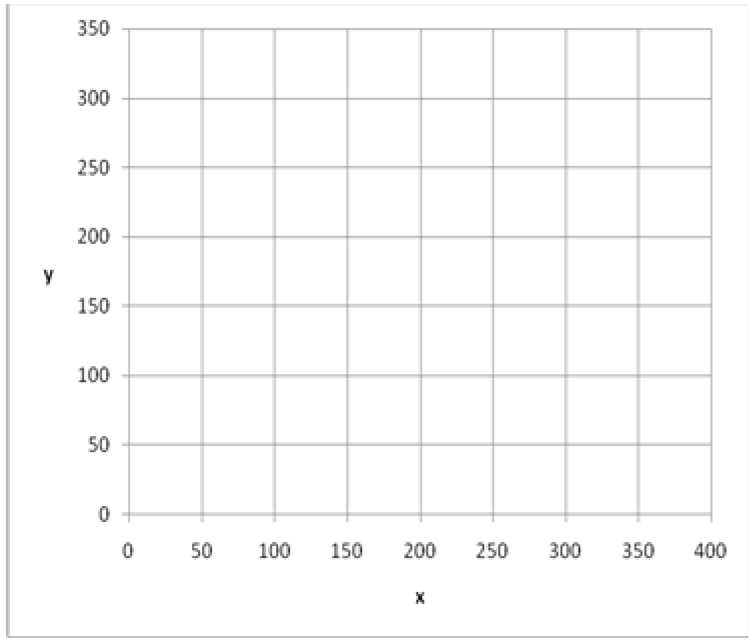
A linear function defined on a nonempty bounded feasible region has a maximum and a minimum value. These value happen at corner points.

Try the following:

2 p. 268

4 p. 268

28 p. 270



Play Section 4.2 Exercises 1

Homework: Review Exercises (pages 307-309) Try as many of 1-14,36,37 as needed in order to feel comfortable with the material.

Chapter Nine: Derivatives

Play Section 9.1 Discussion 1

Section 9.1: Limits

The **Limit** of $f(x)$ as x approaches c is L ($\lim_{x \to c} f(x) = L$) means that as x gets infinitely close to c (but not equal to c) from both sides of c, $f(x)$ approaches a unique real number L.

$\lim_{x \to 2} x^2$

x	1.99	1.999	1.9999	2.0001	2.001	2.01
f(x)	3.9601	3.996001	3.9996	4.0004	4.004001	4.0401

Try the following:

12 p. 546 Note: $f(-0.5)$ does not exist

x	-.51	-.501	-.5001	-.4999	-.499	-.49
f(x)	1.9802	1.9880	1.9998	2.0002	2.0020	2.0202

Play Section 9.1 Exercises 1

Section 9.2: Continuous Functions: Limits at Infinity
Play Section 9.2 Discussion 1
$f(x)$ is **Continuous** at $x=c$ if and only if $\lim_{x \to c} f(x) = f(c)$.

$f(x) = x^2$

$f(x) = \frac{|x|}{x}$

A polynomial function is continuous everywhere.

Section 9.3: Average and Instantaneous Rates of Change: The Derivative
Play Section 9.3 Discussion 1
Find the slope of a tangent line for a curve:

Draw a **Secant Line** (connects any two points on a graph) between points P and Q; one can find the slope. Move Q closer to P, look at the slope. The limit slope as Q gets infinitely close to P is the slope of the tangent line at P. It is also called the **Slope of the curve f(x)** at P.

$$m_{PQ} = \frac{f(x+h) - f(x)}{x+h-x} = \frac{f(x+h) - f(x)}{h}$$

As Q gets closer to P, h gets closer to 0, so

$$m_{\text{tangent line at P}} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

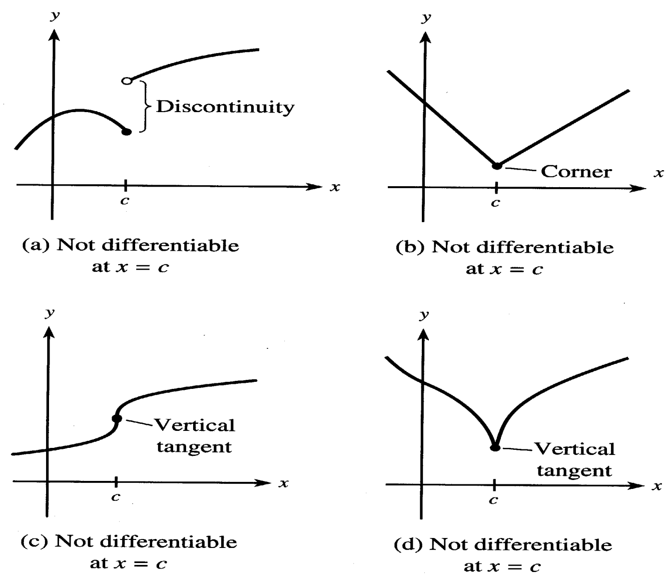
The **Derivative** of f is the function denoted f' and defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ provided the limit exists.

If $f'(x)$ can be found, f is said to be **Differentiable.**

So $f'(x_1)$ is the slope of the tangent line to $y = f(x)$ at the point $(x_1, f(x_1))$.

Notation: $f'(x) = \frac{dy}{dx} = \frac{d}{dx} f(x) = y' = D_x y = D_x f(x)$



If f is differentiable at $x = c$, then f is continuous at $x = c$.

If we are interested in the **Instantaneous Rate of Change** for a function f at a given point, we look at $\Delta t \to 0$. So instantaneous rate of change is $\frac{df}{dt} = f'(t)$.

Let total revenue = $R(x)$. Marginal revenue is the rate of change of $R(x)$ with respect to quantity, so marginal revenue = $R'(x)$.

Find f′(x) if f(x) = x^2

Section 9.4: Derivative Formulas

Play Section 9.4 Discussion 1

Rules for Differentiation

1. If $f(x) = c$, where c is constant, then $f'(x) = 0$.

2. If n is any real number and $f(x) = x^n$, then $f'(x) = nx^{n-1}$.

3. If c is constant and f is differentiable,
$$\frac{d}{dx}\left[cf(x)\right] = cf'(x).$$

4. If f and g are differentiable, then
$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = f'(x) \pm g'(x).$$

Try the following:

2 p. 580

4 p. 580

8 p. 580

20 p. 580

24 p. 580

48 p. 581

Play Section 9.4 Exercises 1

Section 9.9: Applications of Derivatives in Business and Economics

Play Section 9.9 Discussion 1

Let total cost = $C(x)$. Marginal cost is the rate of change of $C(x)$ with respect to quantity, so marginal cost = $C'(x)$.

Let total revenue = $R(x)$. Marginal revenue is the rate of change of $R(x)$ with respect to quantity, so marginal revenue = $R'(x)$.

Let total profit = $P(x)$. Marginal profit is the rate of change of $P(x)$ with respect to quantity, so marginal profit = $P'(x)$.

Try the following:
36 p. 617

Play Section 9.9 Exercises 1

Homework: Review Exercises (pages 620-623) Try as many of 51-60,91,100-104,106,107 as needed to feel comfortable with the material.