Invisible Lattice Points

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Abstract

This talk is about the invisibility of points on the integer lattice $\mathbb{Z} \times \mathbb{Z}$, where we think of these points as (infinitely thin) trees. Standing at the origin one may notice that the tree at the integer lattice point $(1,1)$ blocks from view the trees at $(2,2)$, $(3,3)$, and more generally, at $(n,n)$ for any $n \in \mathbb{Z}_{\geq 0}$. In fact any tree at $(\ell,m)$ will be invisible from the origin whenever $\ell$ and $m$ share any divisor $d$, since the tree at $(\ell/D, m/D)$, where $D = \gcd(\ell,m)$ blocks $(\ell,m)$ from view. With this fact at hand, we will investigate the following questions. If the lines of sight are straight lines through the origin, then what is the probability that the tree at $(\ell,m)$ is visible? Meaning, that the tree $(\ell,m)$ is not blocked from view by a tree in front of it. Is possible for us to find forests of trees (rectangular regions of adjacent lattice points) in which all trees are invisible? If it is possible to find such forests, how large can those forests be? What happens if the lines of sight are no longer straight lines through the origin, i.e. functions of the form $f(x) = ax$. 

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