

## ABSTRACT

Multivariate Functional Principal Component Analysis (MFPCA) is a valuable tool for exploring relationships and identifying shared patterns of variation in multivariate functional data. However, interpreting these functional principal components (PCs) can sometimes be challenging due to issues such as roughness and sparsity. In this dissertation, we establish the theoretical foundations of the penalized MFPCA problem within Hilbert space and propose three novel regularized MFPCA approaches. These approaches utilize eigen decomposition and singular value decomposition (SVD) techniques to enhance the performance of MFPCA by incorporating multiple penalty terms, such as roughness and sparsity penalties.

In the first method, a roughness penalty is directly imposed on functional PCs, extending the eigen decomposition problem to a Hilbert space that specifically accounts for the roughness of the functions. A parameter vector is employed as a tuning parameter to regulate the smoothness of each functional variable. Additionally, this method allows for each functional variable to be smoothed on different domains, providing greater flexibility in handling diverse functional data.

In the other two methods, we establish a mathematical foundation for penalized functional SVD to address the regularized MFPCA problem. Within the functional SVD framework, we propose iterative power algorithms that offer both the flexibility to assign unique tuning parameters for each functional PC and computational efficiency. Moreover, the functional SVD approach allows for the straightforward and simultaneous incorporation of various penalties, such as smoothing and sparsity, each serving a distinct purpose. Additionally, our functional SVD approach introduces an innovative form of sparsity within PC scores, which proves beneficial for obtaining more informative PCs. Similar to the first approach, these two methods also allow each functional variable to be defined on different domains, providing greater flexibility in data analysis.