Impact of Initial Stressor(s) on Cascading Failures in Power Grids

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***Abstract*—Cascading failures in the power grid refer to a chain of events triggered by an initial event such as a single or a combination of failures in a generator, a transmission line, communication or control action. The initial event can be attributed to a natural disaster or an intentional human-made attack. In this paper, cascading failures in the power grid are analyzed under various initial conditions. First, Gaussian, circular and linear stressor(s) are used as the initial events to model the probability of transmission line failure due to the stressor(s). Second, Monte-Carlo simulations are used to analyze the impact of cascading failures in the power grid based on the initial failure patterns. The reported results show that upon the occurrence of an initial triggering event, a combination of parameters (e.g., the number of stressor(s), the number of failed transmission lines in each stressor location, the capacity of the failed transmission lines, the power-grid loading level, the load-shedding constraints at the time of the stressor event) strongly influence the dynamics of cascading failures and may lead to massive blackouts.**

***Index Terms*—Power grid, cascading failure, blackout, stressor event, initial failure, Monte-Carlo simulations.**

I. Introduction

Cascading failures in the power grid are heavily dependent upon the initial stressor event that induces failures in the power grid and initiates a chain of events. A stressor event can be a natural disaster or human-made sabotage attack or error. Power grid parameters such as the number of failed transmission lines, the capacity of the failed transmission lines, the loading level in the power grid, the ability to implement load-shedding, collectively affect the cascading behavior following an initial event. Moreover, the geographical correlation among failures during an initial event can amplify cascading failures [1]–[3]. A combination of parameters determine the initial failures of the power grid due to the occurrence of an initial stressor event and can lead to blackouts of various sizes in the power grid. Therefore, to model the dynamics of the cascading failure in power grid, it is essential to investigate the impact of power grid parameters responses upon an initial stressor event.

In this paper, we study the influence of the initial conditions that conduce the cascading failures in the power grid. We formulate the impact of stressor(s) events analytically using Gaussian, circular and linear degradation functions which result in initial failures in the power grid. We perform simulations on the IEEE 118-bus and IEEE 300-bus topology using power flow simulator to observe the impact of initial stressor(s) event and power grid parameters on cascading failures in the power grid. Simulations show that there is a linear relationship between initial failures in the power grid and stressor(s) intensity. Then, we observe the impact of the number of initially failed transmission lines and the total capacity of the initially failed transmission lines on cascading failures in the power grid. Moreover, we use the power flow simulator to investigate the impact of power grid loading level and load-shedding during initial event. Our simulation results show that a combination of power grid parameters influences cascading failures drastically. These parameters include the number of failed transmission lines, total capacity of the failed transmission lines, number of geographical stressor(s) locations, failed transmission lines in each stressor location, the intensity of the stressor(s), the power grid loading level, the load-shedding constraints. Increasing the values of these parameters during an initial event increases the probability of cascading, i.e., increases the probability of blackout-size in the power grid. Simulation results suggest that the initial condition of the power grid during a stressor(s) event is very crucial; hence, this work paves a way to study and minimize the impact of cascading failures with carefully designing the grid considering these effects.

The paper is organized as follows. A brief description of the related works is given in Section II. In Section III, we introduce modeling of initial failures in a power grid due to a stressor event and show the impact of the stressor event under various scenarios. We analyze the cascading failures in power grid due to the initial failures occurred because of stressor event in Section IV. We conclude and summarize our results in section V.

II. Related Works

In the last two decades, both single and interdependent models were proposed by researchers to capture the cascading failures dynamics in the power grid. Our focus is to study probabilistic models which can be further categorized to Markov-chain based models [4]–[6], branching processes [7], regeneration theory [8]. These models analyze the cascading failures in power grid based on an initial event. The interdependent system model [9]–[16] capture the interdependency between layers of the power grid (e.g., power grid, communication system, and human-operator response) and analyzes cascading

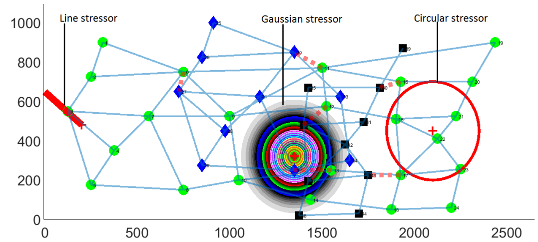


Fig. 1: Gaussian, circular and linear attacks mapped into a 2-D topological space. Failures in the power grid depends on the intensity of the stressor(s). Initial transmission line failures in the power grid calculated after an attack is simulated [1].

failures in the power grid based on interdependent system environment. A data-driven model for simulating the evolution of transmission line failure in power grids is proposed in [17]. Although failures in the communication layer and human operator responses are crucial in cascading failure analysis, we ignored their effects in this paper to simplify our analysis. Bernstein *et. al.* analyzed the power grid vulnerability due to geographically correlated failures in [3]. Impacts of operating characteristics on the sensitivity of the power grids to cascading failures are studied in [18]. In [19], the authors studied the impact of topology in power grids. In [1], the authors analyze the impact of various initial failures on physical infrastructures (e.g., communication networks).

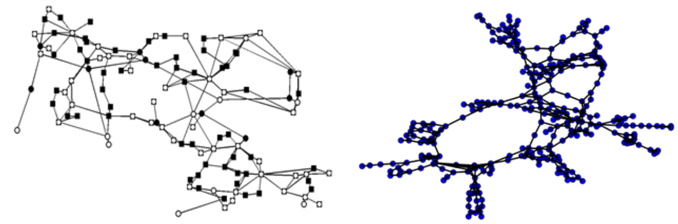
In recent years, researchers contributed significantly to model the cascading failures in power grid. To the best of our knowledge, most of the works done on the probabilistic modeling of cascading failures consider arbitrary initial failures and then focus on modeling the propagation of failures. However, fewer efforts are made to observe the impact of various initial conditions that lead to cascading failures, which is the crucial contribution of this paper. We map the intensity of stressor(s) events with failures in the power grid. No notable extensive analysis has been done to show the correlation between the status of power-grid parameters during an initial stressor(s) event and failures in the power grid that leads to cascading failures. Our work can map the correlation between an initial stressor event and cascading failures in the power grid; thus, this work can investigate cascading failure behavior of the power grid more realistically compared to other works.

III. Modeling the Initial Failures Due to the Stressor(s) and Impact of Stressor(s) on Cascading Failures in Power Grid

In this section, we map the initial transmission line failures in the power grid with stressor intensities.

A. Modeling the initial failures due to stressor(s)

Multiple stressor(s) can occur in one geographical location, or they can spread over different geographical areas. These stressor(s) events can range from natural disasters (e.g., tornado, cyclone, earthquake) to intentional human-made attacks (e.g., use of weapons of mass destruction (WMDs), High

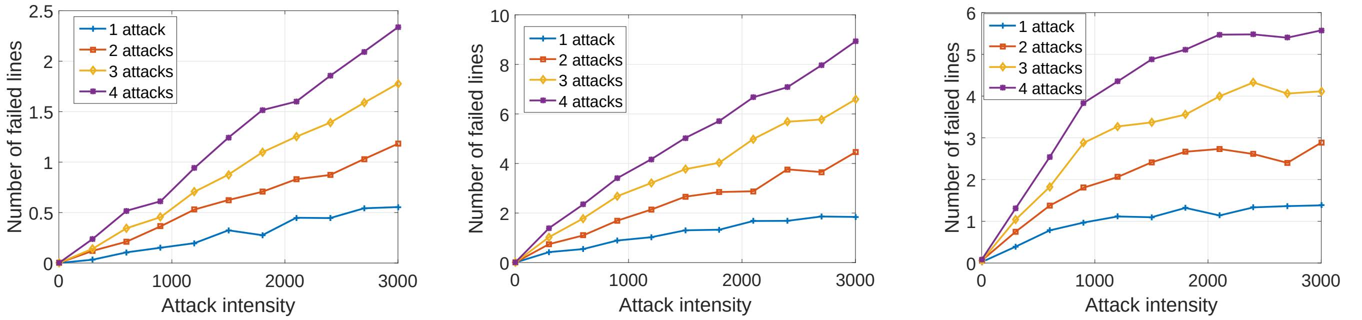


(a) IEEE 118-bus topology (b) IEEE 300-bus topology

Fig. 2: IEEE 118-bus and 300-bus topology

altitude electromagnetic pulses (HEMPs), cyber-attack in the communication layer of the power grid. These events can lead to initial disturbances in the power grid which may include the transmission line failures, generator loss or failures in the communication system. These initial failures can act as a trigger for initiating cascading failures in the power grid. In this paper, we have used spatially-homogeneous stressor(s) centers, which enables us to model multiple stressor(s) events at the same time. The spread of these stressor(s) can vary depending on the intensity of the stressor(s). We use Gaussian, circular and linear degradation functions, which can reasonably characterize various real-world stressor(s) [1]. The intensity of the Gaussian stressor degrades according to the Gaussian function as the spatial distance from the location of occurrence increases. The intensity of the function has the peak at the mean of the degradation function. Two parameters entirely describe a circular degradation function: radius of the circle (r) and the intensity of the stressor at the center (I). The main difference between a Gaussian and a circular stressor is in their degradation function. For a Gaussian stressor, the intensity of the stressor degrades with e^{-d^2} while for the circular stressor; it degrades with $1/d^2$. For the Gaussian case, d is the minimum distance from the stressor center to the point where intensity needs to be measured (e.g., Bus location, transmission line fault). Similarly, for the circular case, d is the distance of from the stressor center to the point of intensity measure. Linear stressor(s) can be used to model natural disasters like tornadoes, which can occur in any geographical location with a shallow radius but having almost equal strength over the region it spreads. Figure 1 shows a realization of these three types of degradation functions over a physical infrastructure. It is evident that attacks with the same intensity can lead to a different impact on the power grid (e.g., different transmission line failures) depending on the nature of the attack.

We denote the stressor(s) event by W and the stressor intensity at any point (x_i, y_i) from the center of the stressor(s) with $I_w(x_i, y_i) \geq 0$ (attack intensity is either zero or a positive number and cannot be negative). The shape of a stressor can be either Gaussian, circular, linear or a combination of any of these over the power grid topology. The stressor intensity degrades with distance from the center. To calculate the probability of line failures due to a stressor event, we divide each of the power grid transmission lines into *N* points (*N* can



(a) For Gaussian stressor(s)

(b) For circular stressor(s)

(c) For linear stressor(s)

Fig. 3: Average number of failed transmission lines in IEEE 118-bus topology due to to Gaussian, circular and linear stressor(s) with various intensities.

be infinity large, i.e., the distance between two adjacent points can be close to zero) and measure the stressor intensity at those points after the occurrence of a stressor event. We then take the maximum intensity calculated in those N points. We assume that if the maximum intensity at any point over the line crosses a certain threshold, then the line will fail. Here, we assume N to be sufficiently large. An alternative approach of calculating the maximum stressor intensity on a transmission line can be to calculate the minimum distance between the transmission line and the stressor center. Since the stressor intensity degrades over distance, it is intuitive that minimum distance from the stressor center would result in maximum intensity; with the peak intensity being at the center of the stressor(s). Hence, the maximum stressor intensity on a transmission line would be inversely proportional to the minimum distance between the transmission line and the stressor center. For a single stressor event occurred in a geographical location, we define the failure probability of a transmission line as:

$$p((B_i, B_j)|W = w) = \min\left(\max_{k \in 1,\dots,N} I_w(x_k, y_k), 1\right), \tag{1}$$

where $p((B_i, B_j)|W = w)$ denotes the failure probability of a transmission line of the power grid, (B_i, B_j) is the transmission line from B_ith bus to B_jth bus, and (x_k, y_k) is the location of the kth point on (B_i, B_j). For multiple stressor events occurring at the same time, the total stressor intensity at (x_k, y_k) is

$$p((B_i, B_j)|W = (w_1, \dots, w_L)) = \min\left((\sum_{i=1}^{L} \max_{k \in 1,\dots,N} I_{w_i}(x_k, y_k)), 1\right), \tag{2}$$

where L denotes the number of stressors.

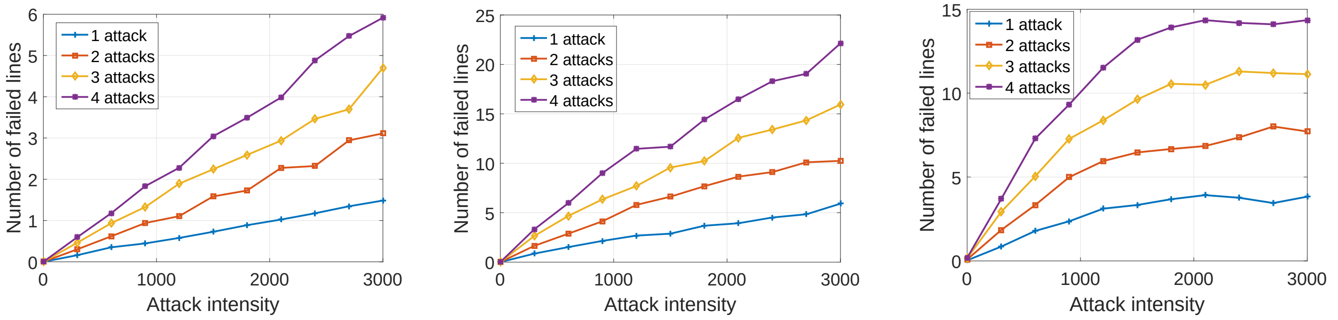
We calculate the total number of failed transmission lines in the power grid due to the occurrence of the stressor(s) using the measured individual transmission line probability. Similarly, we can calculate the bus (node) failure probability due to multiple stressor events using the following equation

$$p((B_i)|W = (w_1, \dots, w_L)) = \min\left((\sum_{i=1}^{L} I_{w_i}(x_k, y_k)), 1\right) \tag{3}$$

Now, considering the fact that initial failures of a network component does not depend on other components [2], the joint failure probability of the power grid transmission lines due to stressor(s) event can be represented using the product of their individual failure probabilities. Therefore, for a power grid with M transmission lines we have

$$p\Big(((B_1, B_2), \dots, (B_{M-1}, B_M))|W = w\Big) = \prod_{(B_i, B_j) \in V} p((B_i, B_j)|W = w), \tag{4}$$

where V is the collection of all transmission lines in the power grid. Depending on the geographical position and the intensity of the stressor(s), we obtain different initial transmission lines failures. Figure 3 and Fig. 4 shows a plot of the average number of failed transmission lines due to the stressor(s) with Gaussian, circular and linear degradation functions with various intensities. We obtain the average number of failed transmission lines using Monte-Carlo simulations over the IEEE 118-bus topology (186 transmission lines, Fig. 2(a)) and IEEE 300-bus topology (411 transmission lines, Fig. 2(b)) with 1000 sample realizations . In each sample realization, we generate stressor(s) at random locations (uniformly distributed) and calculate the intensity of stressor at every bus and transmission line using (1) and (2). Then we take the expectation of transmission line failures over the total realizations with a stressor(s) intensity for the three degradation functions. In both IEEE 118-bus and 300-bus cases, we can see that the expected number of failed transmission lines increases linearly with the increase in stressor(s) intensity. Again, it can be observed that for a particular stressor type and same attack intensity, for IEEE 300-bus system we get higher average failed lines compared to IEEE 118-bus system. Aforementioned is because, for IEEE 300-bus system, node density over the geographical region is higher compared to IEEE 118-bus



(a) For Gaussian stressor(s) (b) For circular stressor(s) (c) For linear stressor(s)

Fig. 4: Average number of failed transmission lines in IEEE 300-bus topology due to to Gaussian, circular and linear stressor(s) with various intensities.

system. From Fig. 3 and Fig. 4, it is visible that with same stressor intensity, circular stressor creates the worst impact on the both the IEEE 118-bus and IEEE 300-bus topology. On the contrary, Gaussian stressor has the least impact since Gaussian stressor(s) intensities decay at a faster rate (e^{-d^2})compared to a circular stressor(s) which degrades with $1/d^2$ where $d < r$.

As shown above, the expectation of transmission line failure in the power grid has a linear relationship with stressor(s) intensities, i.e., the number of line failure increases linearly with the stressor(s) intensity for Gaussian, circular and linear degradation functions. With this relationship at hand, we now have a model that is capable of giving the initial failures in a power grid due to a stressor(s) event with various intensities. This is important because now we can predict the impact of a real-world natural disaster or human-made attacks. In the next section, we will use the obtained initial failures due to stressor(s) in optimal power flow simulator (MATPOWER) [20] and analyze cascading failures in power grid.

B. Power-Flow Optimization Framework

We use MATPOWER [20], a package of MATLAB m-files for solving the DC power flow optimization problem and used it in our cascading failures analysis. It uses power-flow distribution framework and can give overloaded transmission lines; which was used in several previous works [5], [6], [21]. We skip the detail description of MATPOWER tool here for space constraint. Interested readers can review the MATPOWER manual for a complete understanding of the simulation tool. We consider a line failure when power flow through a line exceeds maximum allowable capacity through that line. Once we find an overflow in a transmission line, we fail that line and re-calculate optimal power flow (OPF) using the remaining transmission lines. In our simulations, we take one transmission line failure at a time. If multiple transmission lines exceed the capacity threshold, we fail the line with maximum capacity. We take 1000 random realizations and calculate associated transmission line failure probabilities due to the stressor(s) using (1) and (2). We use the same intensity of the stressor(s) for one turn of 1000 realizations and calculate the average number of failed transmission lines.

C. Impact of stressor(s) event on cascading failures

We use Gaussian, circular and linear stressor(s) over the IEEE 118-bus topology (Fig. 2(a)), and consider these stressor(s) as initial events that may lead to cascading failures in the power grid. We only show the results using the IEEE 118-bus topology here for space constraint. We perform Monte-Carlo simulation to analyze the impact of the stressor(s) on cascading failures in power grid based on OPF analysis. Since transmission line failures increase linearly with stressor intensity, one stressor event can generate multiple transmission line failures if the stressor intensity is high. However, the line failures will exhibit clustering (failed lines will be close to each other). On the contrary, multiple stressors can initiate multiple failures, and the stressor locations can be distributed randomly (inhibition). Here, we define that a cascading failure event occurred if more than five percent additional transmission lines are failed after an initial stressor(s) event. If, $F_{threshold}$ is the threshold for a cascading event, $F_{initial}$ and M are the number of initially failed transmission lines and total number of transmission lines respectively then, $F_{threshold} = F_{initial} + 0.05 * M$. For a realization if the total number of line failure exceeds $F_{threshold}$, we consider that as a cascading failure event. For example, if three transmission line fails due to a stressor(s) event, then we say a cascading failure event occurred if more than twelve transmission line fails for the IEEE 118-bus case, which has 186 transmission lines. From Fig. 5 it is visible that inhibition of failures generates more cascading failure event than clustering, i.e., if the transmission line failures are randomly distributed, then there is a higher likelihood of cascading failures in the power grid. The reason for low cascading due to clustering is that the power grid has a better control mechanism to mitigate the impact of localized failures using load-shedding or islanding, as the location of the failed lines are very close to each other. Most of the probabilistic models consider random failures distributed over the power grid [5], [6], [13]. However, if the failed transmission lines are distributed (can be the result from multiple stressor events occurring at the same time in various locations), that in turn increases the probability of cascading

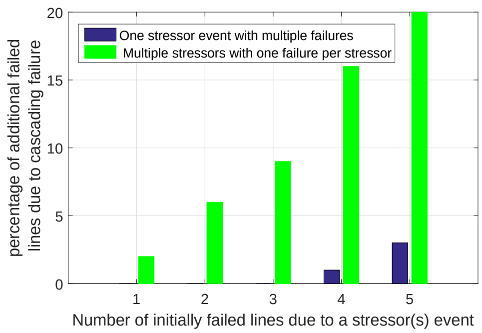


Fig. 5: Number of failed transmission lines when one stressor location with multiple failures (blue) and considering randomly distributed failed transmission lines (green) where a stressor event contribute one transmission- line failure (we pick the line with maximum intensity to fail).

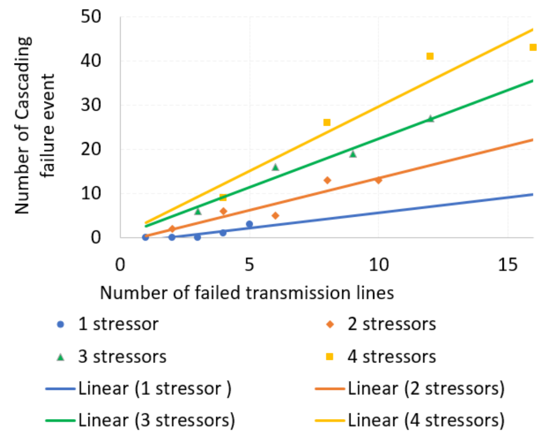


Fig. 6: Number of cascading failure event in power grid with different number of attack points and number of transmission line failures.

failure in power grid.

Figure 6 shows the simulation result for attacks with multiple transmission line failures. We can see that for the same number of transmission line failures, if we increase the number of attack points, power grid becomes more cascade-prone than the previous case. Here, in Fig. 6, we use linear curve fitting (blue, red, green, and orange lines represent various stressor(s)) to show the impact of inhibition clearly.

IV. Impact of Initial Failures Due to a Dtressor Event in Cascading Failures

We now apply our initial failure model in MATPOWER OPF simulator to calculate the impact of stressor(s) events on cascading failures in power grid. Simulations using the other IEEE topologies follow the same pattern.

A. Impact of number of failed transmission lines and capacity of the failed transmission lines

We define percentage of additional transmission lines lost due to the cascading failures as $\Delta_M/(M - M_{initial})$, where Δ_M = additional transmission lines lost due to cascading; M = total transmission lines of the power grid; $M_{initial}$ = number of transmission lines failed due to initial event. Similarly, percentage of additional capacity lost due to the cascading failures as $\Delta_C/(C_{total} - C_{initial})$, where Δ_C = additional capacity lost due to the cascading; C_{total} = total capacity of the power grid; $C_{initial}$ = total capacity of the initially failed lines. Figure 7 represents the impact of various initially failed transmission lines of fixed total capacity and the total capacity of the failed transmission line during an initial event using OPF simulations. In Fig. 7(a), we keep the total capacity of the failed lines as constant and then increase the number of failed transmission lines. We take randomly distributed line failures for 1000 samples in each case. These initial line failures are generated using random stressor events over the IEEE 118-bus topology. Our simulation results suggest that, if the total capacity of the failed lines is fixed, increase in the number of line failures makes the power grid more cascade-prone. In Fig. 7(b), a similar type of simulation is done with a fixed number of failed transmission lines (randomly chosen from the 186 lines) while varying the total capacity of the failed lines. The results suggest that the percentage of additional capacity lost due to cascading failures increases if the total capacity of the initially failed lines is increased. Thus, we conclude both numbers of initial line failures and total capacity of the failed lines during a catastrophic event can lead to the cascading failures in power grid.

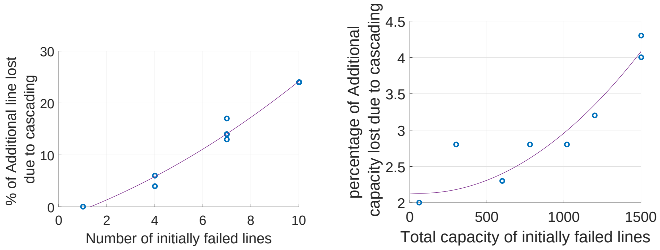
B. Impact of power grid loading level and load-shedding constraint on cascading failures in power grids

Power-grid loading level, $l \in$ [0,1] is defined as the ratio of the total demand and the generation-capacity of the power grid. The ratio of the uncontrollable loads (loads that do not participate in load shedding) and the total load in the power grid is termed the load-shedding constraint, denoted by, $\theta \in$ [0,1]. Here, the stress of the power grid increases as we increase l, and $\theta = 0$ implies no load shedding constraint while $\theta = 1$ indicates no load shedding can be implemented.

To observe the impact of l and θ, we consider fixed number of initial transmission line failures in our simulation. We observe that when the power grid is highly stressed it is more cascade-prone than when the grid is nominally stressed. Figure 8 shows a linear relationship between average number of failed transmission lines and the operating parameters. We can also observe that there is a critical operating point for both $l$and θ (approximately 0.8 and 0.2 for l and θ in our case). We observe a sharp increase in average cascading failures beyond this critical parameter setting. Similar observations were found in [5], [6].

V. Conclusion

In this paper, we analyze the impact of initial stressor event that leads to cascading failures. We have formulated the initial failures in power grid with various attack types (Gaussian, circular and linear) and simulate using IEEE 118-bus and



(a) varying number of initially failed lines (b) varying total capacity of the initially failed lines

Fig. 7: Relationship between number of initially failed transmission line due to a stressor event with percentage of additionally failed lines due to cascading when the total capacity of the failed transmission lines are fixed, and the total capacity of the initially failed transmission lines with additional capacity lost due to cascading when the number of the failed transmission lines are fixed.

300-bus topology. Our simulations suggest that the number of initially failed transmission lines are linearly proportional with attack intensity. We observe that cascading failures in the power grid is correlated with different power grid parameters during an initial stressor(s) event. These parameters include transmission line failures, the capacity of the failed transmission lines, number of stressor locations, power grid operating parameters such as power grid loading level, load-shedding constrain during an initial stressor event. All these initial conditions eventually determine the blackout-size during a cascading event. Although several models can be found analyzing cascading failures in power grid, most of them consider arbitrary initial conditions to model the cascading failure behavior. Our work captures the impact of initial conditions during a stressor(s) event and analyzes cascading failures phenomenon from the stressor event occurred. Future works may include capturing the impact of continuous time-varying degradation functions and identify critical operating settings for the power grid for such degradation functions.

VI. Acknowledgement

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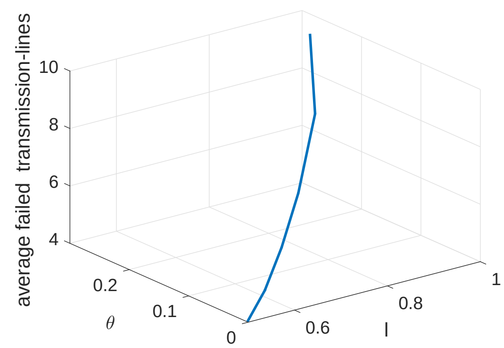

Fig. 8: Dependence of the average number of failed transmission lines on power grid operating parameters (l and θ).

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